

# NUMERICAL METHODS IN COMPUTATIONAL ENGINEERING

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Recommended bibliography

1. **NUMERICAL METHODS FOR ENGINEERS AND SCIENTISTS**, Joe D.Hoffman, Purdue University, Dept. of Mechanical Engineering Taylor & Francis, London, New York, Singapore, 2001.

Emphasizing the finite difference approach for solving differential equations, the second edition of Numerical Methods for Engineers and Scientists presents a methodology for systematically constructing individual computer programs. Providing easy access to accurate solutions to complex scientific and engineering problems, each chapter begins with objectives, a discussion of a representative application, and an outline of special features, summing up with a list of tasks students should be able to complete after reading the chapter- perfect for use as a study guide or for review. The AIAA Journal calls the book "...a good, solid instructional text on the basic tools of numerical analysis."

The objective of this book is to introduce the engineer and scientist to numerical methods which can be used to solve mathematical problems arising in engineering and science that cannot be solved by exact methods. With the general accessibility of high-speed digital computers, it is now possible to obtain rapid and accurate solutions to many complex problems that face the engineer and scientists.

The approach taken is as follows:

1. Introduce a type of problem.
2. Present sufficient background to understand the problem and possible methods of solution.
3. Develop one or more numerical methods for solving the problem.
4. Illustrate the numerical methods with examples.

In most cases, the numerical methods presented to solve a particular problem proceed from simple method to complex methods, which in many cases parallels the chronological development of the methods. Some poor methods and some bad methods, as well as good methods, are presented for pedagogical reasons. Why one method does not work is almost as important as why another method does work.

The material in the book is divided into three main parts:

- I. Basic tools of Numerical Analysis
- II. Ordinary Differential Equations
- III. Partial Differential Equations.

Part I considers many of the basic problems that arise in all branches of engineering and science. These problems include: solution of systems of linear algebraic equations. Eigenproblems, solution of nonlinear equations, polynomial approximations and interpolation, numerical differentiation and differential formulas, and numerical integration. These topics are important both in their own right and as the foundation for Parts II and III.

Part II is devoted to the numerical solution of ordinary differential equations (ODEs). The general features of ODEs are discussed. The two classes of ODEs (i.e. initial-value ODEs and boundary-value ODEs) are introduced, and the two types of physical problems (i.e. equilibrium problems and propagation problems) are discussed. Numerous numerical methods for solving ODEs are presented.

Part III is devoted to the numerical solution of partial differential equations (PDEs). Some general features of PDEs are discussed. The three classes of PDEs (i.e. elliptic PDEs, parabolic PDEs, and hyperbolic PDEs) are introduced, and the two types of physical problems (e.g. equilibrium problems and propagation problems) are discussed. Several model PDEs are presented. Numerous numerical methods for solving the model PDEs are presented.

The material presented in this book is an introduction to numerical methods. Many practical problems can be solved by the methods presented here. Many other practical problems require other or advanced numerical methods. Mastery of the material presented in this book will prepare engineers and scientists to solve many of their everyday problems, give them the insight to recognize when other methods are required, and give them background to study other methods in other books and journals.

2. **NUMERICAL METHODS FOR ENGINEERING APPLICATIONS**, Joel H. Ferziger,  
Stanford University  
John Wiley & Sons, Inc., New York, 1998

State-of-the-art numerical methods for solving complex engineering problems

Great strides in computer technology have been made in the years since the popular first edition of this book was published. Several excellent software packages now help engineers solve complex problems. Making the most of these programs requires a working knowledge of the numerical methods on which the programs are based. Numerical Methods for Engineering Application provides that knowledge.

While it avoids intense mathematical detail, Numerical Methods for Engineering Application supplies more in-depth explanations of methods than found in the typical engineer's numerical "cookbook." It offers complete coverage of most commonly encountered algebraic, interpolation, and integration problems. Ordinary differential equations are examined in great detail, as are three common types of partial differential equations--parabolic, elliptic, and hyperbolic. The author also explores a wide range of methods for solving initial and boundary value problems.

This complete guide to numerical methods for solving engineering problems on computers provides:

- \* Practical advice on how to select the best method for a given problem
- \* Valuable insights into how each method works and why it is the best choice

- \* Complete algorithms and source code for all programs covered
- \* Code from the book and problem-solving programs designed by the author available from the author's website

Numerical Methods for Engineering Application is a valuable working resource for engineers and applied physicists. It also serves as an excellent upper-level text for physics and engineering students in courses on modern numerical methods.

Table of content

Short Review of Linear Algebra.

Interpolation.

Integration.

Ordinary Differential Equations: I. Initial Value Problems.

Ordinary Differential Equations: II. Boundary Value Problems.

Partial Differential Equations: I. Parabolic Equations.

Partial Differential Equations: II. Elliptic Equations.

Partial Differential Equations: III. Hyperbolic Equations.

Appendices.

Index.

3. **NUMERICAL METHODS IN ENGINEERING AND SCIENCE**, Carl E. Pearson, University of Washington  
Van Nostrand Reinhold Company, New York, 1986

A course in numerical analysis has become accepted as an important ingredient in the undergraduate education of engineers and scientists. *Numerical Methods in Engineering and Science* reflects experience in teaching such a course for several years. Related work in industry and research has influenced my choice of content and method of presentation.

Most student at the undergraduate level will have had, at the very least, an introductory course in ordinary differential equations. Tutorial appendices on complex variable, determinants, partial differentiation, and Taylor expansions are included at the end of this book. Other background material is developed as needed. For example, Chapter 2 (linear equations) begins with one outline of matrix algebra; this will represent review material for some students, but it will spare others the necessity of consulting references. Proofs for almost all results of importance are given, in what I hope is palatable form. Overall, the book should be reasonably self contained.

A number of illustrative computer programs are provided. The language chosen is FORTRAN (ANSI 77) because of the wide availability of service and application programs in that language. I am aware of course that some readers, for good reasons, will prefer other languages; however, since most languages are sufficiently similar to FORTRAN there should be little difficulty in translating from one language to another as required.

I have not tried to include an extensive collection of library-type programs. At this stage, it seems to me that it is important for the student to acquire facility in writing actual programs (and this is called for, in the text and exercises). This kind of programming experience should help solidify the understanding of numerical techniques, as well as provide perspective for the eventual use of subroutine libraries.

This book contains more material than would be normally included in an introductory undergraduate course. I feel, however, that it is useful to cover the various topics with some degree of completeness so that the book may serve the student as a subsequent reference and source of ideas. Illustrative examples are given throughout the text. It is important for the student to work the problems, and a fairly large number of problems that illustrate (and in some cases, extend) the material of the text will be found at the end of each chapter. In a course based on this book, the instructor may want to supplement these problems with some of the usual kind of drill-type exercises.

4. **ADVANCED MATHEMATICAL METHODS FOR ENGINEERING AND SCIENCE STUDENTS**, G. Stephenson and P.M. Radmore, Imperial College London, University College, London Cambridge Univ. Press, 1999

This book provides a solid foundation to a number of important topics in mathematics of interest to science and engineering students. The authors' approach is simple and direct, the emphasis being on the analytical structure and applications of the material. The text is virtually self-contained, assuming only that the student has received a good basic course in ancillary mathematics. Each chapter contains a large number of worked examples, and concludes with problems for solution, with answers given in the back of the book. There is no comparable text that covers this material in such a concise form. This book will be of great value to undergraduates in physics, chemistry, theoretical biology, and in all engineering disciplines, as a source book of advanced mathematical methods, and also to postgraduate students as a revision text.

#### Contents

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1. Suffix notation and tensor algebra; 2. Special functions; 3. Non-linear ordinary differential equations; 4. Approximate solutions of ordinary differential equations; 5. Contour integration; 6. Applications of contour integration; 7. Laplace and Fourier transforms; 8. Partial differential equations; 9. Calculus of variations.

5. **THE FINITE ELEMENT METHOD – LINEAR STATIC AND DYNAMIC FINITE ELEMENT ANALYSIS**, Thomas J.R. Hughes, Stanford University, Dover Publications, New York, 2000

This book is primarily intended for the graduate level, although advanced undergraduates will find much of it accessible. An undergraduate degree in engineering, mathematics, computer science, or any of the physical sciences constitutes essential background. Courses in applied linear algebra and elementary ordinary and partial differential equations are desirable prerequisites. A working knowledge of FORTRAN (the dominant language used in finite element programming) is also necessary for understanding the software presented.

The book emphasizes heat conduction and elasticity as primary vehicles for developing finite element method because of the widespread interest in these theories in the applied sciences. By virtue of the fact that the partial differential equation of heat conduction, namely, the Laplace/Poisson equation, appears

under different names in virtually all branches of engineering and physics, most students have had some familiarity with it. Some exposure to the theory of elasticity is also desirable. This can be obtained, for example, in a good advanced course on strength of materials. Students at Caltech and Stanford frequently have taken courses in elasticity simultaneously with finite elements. Background in structural mechanics (i.e., the theory of beams, plates, and shells) is certainly an asset when it comes to studying this book but is not essential. It is worth noting that students who have taken this finite element course have had very diverse backgrounds (e.g. geophysics, chemical engineering, planetary sciences, coastal engineering, electrical engineering, computer science, mathematics, bioengineering, aeronautics and astronautics, material science, physics, earth sciences, environmental engineering, biomechanics, thermo sciences, engineering design, applied mechanics, earthquake engineering, and, of course, mechanical and civil engineering). In this spirit the book emphasizes fundamental finite element concepts and techniques applicable to a very broad range of problems and thus constitutes a suitable text for most students in the physical sciences.

It is my experience that most finite element methodology can and should be taught initially within the confines of linear behavior before introducing nonlinear effects. A solid understanding of finite elements in linear analysis is, of course, very useful in its own right. In addition, it makes the subject of nonlinear finite element analysis much more accessible.

A linear static and dynamic analysis computer program based on the methods developed in the book is also presented. The program is named DLEARN and it contains a very complete library of finite element software tools. This program is suitable for homework assignments, projects (e.g., programming additional elements), and research studies. DLEARN is highly structured for readability, maintainability, and extendibility and has been written specifically to complement and enhance the procedures described in the remainder of the book.

**6. MATHEMATICAL METHODS IN PHYSICS AND ENGINEERING WITH MATHEMATICA**, Ferdinand F. Cap,  
Chapman & Hall/CRC, London, 2003

More than ever before, complicated mathematical procedures are integral to the success and advancement of technology, engineering, and even industrial production. Knowledge of and experience with these procedures is therefore vital to present and future scientists, engineers and technologists.

Mathematical Methods in Physics and Engineering with Mathematica clearly demonstrates how to solve difficult practical problems involving ordinary and partial differential equations and boundary value problems using the software package Mathematica (4.x). Avoiding mathematical theorems and numerical methods-and requiring no prior experience with the software-the author helps readers learn by doing with step-by-step recipes useful in both new and classical applications.

Mathematica and FORTRAN codes used in the book's examples and exercises are available for download from the Internet. The author's clear explanation of each Mathematica command along with a wealth of examples and exercises make Mathematical Methods in Physics and Engineering with Mathematica an outstanding choice both as a reference for practical problem solving and as a quick-start guide to using a leading mathematics software package.

7. **PARTIAL DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS WITH MATHEMATICA**, Prem K. Kythe, Pratap Puri, Michael R. Schäferkötter,  
University of New Orleans,  
Chapman & Hall/CRC, London, 2003

- Incorporates extensive revisions, including the use of the latest version of Mathematica (4.x)
- Extends the topics addressed to include initial and boundary value problems from continuum mechanics as well as mathematical physics
- Provides the underlying mathematical analysis with focus on the development of methods
- Includes answers to all exercises with hints and detailed solutions to some of the more difficult ones

Furnishes Mathematica programs on the Internet at [www.crcpress.com](http://www.crcpress.com)

Early training in the elementary techniques of partial differential equations is invaluable to students in engineering and the sciences as well as mathematics. However, to be effective, an undergraduate introduction must be carefully designed to be challenging, yet still reasonable in its demands. Judging from the first edition's popularity, instructors and students agree that despite the subject's complexity, it can be made fairly easy to understand.

Revised and updated to reflect the latest version of Mathematica, *Partial Differential Equations and Boundary Value Problems with Mathematica*, Second Edition meets the needs of mathematics, science, and engineering students even better. While retaining systematic coverage of theory and applications, the authors have made extensive changes that improve the text's accessibility, thoroughness, and practicality.

New in this edition:

- Upgraded and expanded Mathematica sections that include more exercises
- An entire chapter on boundary value problems
- More on inverse operators, Legendre functions, and Bessel functions
- Simplified treatment of Green's functions that make it more accessible to undergraduates
- A section on the numerical computation of Green's functions
- Mathematica codes for solving most of the problems discussed
- Boundary value problems from continuum mechanics, particularly on boundary layers and fluctuating flows
- Wave propagation and dispersion

With its emphasis firmly on solution methods, this book is ideal for any mathematics curricula. It succeeds not only in preparing readers to meet the challenge of PDEs, but also in imparting the inherent beauty and applicability of the subject.

8. **NUMERICAL METHODS IN ENGINEERING WITH MATLAB**, Jaan Kiusalaas  
The Pennsylvania University,  
Cambridge Univ. Press, New York, 2005

Numerical Methods in Engineering with MATLAB® is a text for engineering students and a reference for practicing engineers, especially those who wish to explore the power and efficiency of MATLAB®. Examples and applications were chosen for their relevance to real world problems, and where numerical solutions are most efficient. Numerical methods are discussed thoroughly and illustrated with problems involving both hand computation and programming. MATLAB® mfiles accompany each method and are available on the book web site. This code is made simple and easy to understand by avoiding complex bookkeeping schemes, while maintaining the essential features of the method. MATLAB® was chosen as the example language because of its ubiquitous use in engineering studies and practice. Moreover, it is widely available to students on school networks and through inexpensive educational versions. Explore numerical methods with MATLAB®, a great tool for teaching scientific computation.

Only available MATLAB® based engineering numerical methods text. Includes numerous problems and examples Each method is accompanied by computer code written in MATLAB® which is also available on the book web site

#### Contents

1. Introduction to MATLAB®; 2. Systems of linear algebraic equations; 3. Interpolation and curve fitting; 4. Roots of equations; 5. Numerical differentiation; 6. Numerical integration; 7. Initial value problems; 8. Two-point boundary value problems; 9. Symmetric Matrix Eigenvalue problems; 10. Introduction to optimization; Appendices.

#### 9. **OBJECT-ORIENTED METHODS AND FINITE ELEMENT ANALYSIS**, R.I. Mackie, University of Dundee, Saxe-Coburg Publications, 2000

The finite element method was first used in 1956 with the pioneering work of Clouth et al [M.J. Turner, R.W. Clough, H.C. Martin and L.J. Topp, "Stiffness and deflection analysis of complex structures", J. Aeronaut. Sci., vol. 23, 805-23, 1956.], and since then it has found ever wider application in engineering. Its primary applications are still in the field of structural engineering, but other fields such as fluids, hydraulics, environmental problems, electro-magnetics etc are increasingly utilizing the method. The beauty of the method is that the same basic procedure can be applied to an enormous range of problems. Essentially, the domain of interest is split up into a number of small regions called elements. In two dimensions these regions are usually triangular or quadrilateral in shape, in three dimensions they may be tetrahedrons or bricks. For each of these regions a matrix,  $k$ , is obtained which describes, in an approximate manner, the behaviour of that region. In structures  $k$  represents the stiffness of the region and is called the stiffness matrix. The accuracy with which  $k$  represents the stiffness of the region is dependent upon the size and complexity of the element. The stiffness matrices of each element are assembled together to obtain a large stiffness matrix,  $K$ , representing the stiffness of the whole structure. A large number of commercial finite element packages are available on the market, and literally billions of pounds have been invested in finite element software.

There have been, and continue to be, many developments in finite elements technology over the years. Various types of elements have been developed to

handle different situations ranging from simple one dimensional problems to shell structures and fully three dimensional elements. Special elements exist for fracture mechanics, contact problems, and for dealing with foundations. There continues to be much research carried out into handling different sorts of non-linearity, and the search for ever more effective solution procedure continues.

An every finite element practitioner knows, an enormous amount of time, and not a little frustration, can be expected on data entry, in particular on mesh generation. In the early days of finite elements analysis data entry was accomplished by means of data files. Now finite element packages have graphical user interfaces seeking to make the data entry, and results analysis, as straight forward as possible. An essential part of mesh generation is algorithms that enable good meshes to be generated with as little input from the user as possible.

When finite element analysis was in its infancy large and expensive computers were required in order to implement the finite element method. Nowadays computers are everywhere, and very powerful desktop machines are cheaply available. There have also been vast improvements in the ease of use of finite element software. All this is leading to increased accessibility of the finite element method as an everyday engineering tool. From being an extremely specialized aspect of analysis, the finite element method is becoming a much more integrated part of the whole design process. One example of this is developments that allow links between CAD models and finite element models. These sorts of links, together with links between finite element software and other programs and databases are likely to increase.

All these developments have greatly increased the accessibility and usability of finite element software, but this has been coupled with a vast increase in software complexity.

The ever increasing complexity of software is a problem with which the whole of the software engineering world is faced. In recent years one of the key developments in programming, design and analysis has been the introduction of object-oriented methods. In traditional programming languages the focus is on the data flow, but in the object-oriented approach data and functions are linked together into single entities called objects. The most widely used object-oriented language is C++, with Java also being extremely popular, particularly in Web based applications. Object-oriented languages allow the software to more closely represent the "real world" through their greater data modeling capabilities. This makes them more suitable for handling complexity, and, at least in theory, leads to reduced software maintenance and development costs.

Most finite element software was developed in Fortran, and a great deal of time and money has been invested in this software. It might therefore be asked whether is sensible to seek to apply a whole new programming paradigm. However, it is instructive to consider the sources of complexity in finite element programming:

- The many and varied types of elements,
- The vast range of solution algorithms, and complexity inherent in them,
- Utility algorithms such as mesh generation and node ordering routines,
- Graphical user interfaces,
- Links with other programs and databases,
- The need for ever increasing integration with the overall design process.

It should be clear from this that finite element programming is about much more than merely implementing a set of numerical algorithms, though that in

itself could be challenging enough. Finite element programming is about software engineering, and needs to be treated as such. It is also worth noting that it is currently intended that the next release of Fortran will include object-oriented features. Furthermore, at least one large commercial code has been rewritten in C++. It is therefore entirely appropriate to consider how the object oriented approach might be applied to finite element programming, and that is what this book is about. Writing object oriented finite element software is much more than merely rewriting a Fortran program in C++; it requires a different approach to the program, both in its overall design, and programming details. This book seeks to give insight into the changes required in program design order to produce truly effective object-oriented programs, and to reap the benefits of the object-oriented approach.